

# Teaching in Year 6 – part two

*Addressing the raised expectations in the Key Stage 2 SATs for English and mathematics with strategies for moderating the quality of children's writing at the end of Key Stage 2*

## 7.3 What teachers must assess

ARA

In 2017, teachers must make judgements for each eligible pupil against the standards set out in the [interim teacher assessment frameworks](#)<sup>32</sup> or the [interim pre-key stage standards](#)<sup>33</sup>. [Exemplification material](#)<sup>34</sup> is also available to help teachers make their judgements where they want additional guidance.

Teachers must use their knowledge of a pupil's work over time, taking into account their written, practical and oral classwork.

2017 national curriculum assessments

### Key stage 2

Pre-key stage 2:  
pupils working below  
the test standard

Interim teacher assessment framework  
July 2016

2016 national curriculum assessments

### Key stage 2

2016 teacher assessment  
exemplification:  
end of key stage 2

Reading

Working at the  
expected standard

2016 national curriculum assessments

### Key stage 2

2016 teacher assessment  
exemplification:  
end of key stage 2

Mathematics

Working at the  
expected standard



April 2016



Revised March 2016



# Children working below the test standard

For pupils working below the standard of the tests the pre-key stage standards contain statements which define appropriate expectations reflecting core knowledge and skills these pupils need to progress.

Some of the statements are based on the elements of the KS1 curriculum that remain relevant for these pupils.

**Pre-key stage standards for KS2 reading, writing & mathematics:**

**Foundations for the expected standard**

**Early development of the expected standard**

**Growing development of the expected standard**

| Code | Description  | Notes  |
|------|--|--|
| BLW  | Below the standard of the interim pre-key stage standards    | Pupils submitted as BLW for English reading, English writing and mathematics must also have either P scales or NOTSEN submitted.   |
| PKF  | Pre-key stage - foundations for the expected standard        | If interim pre-key stage standards are submitted when a test result exists the test result will be used in performance tables.   |
| PKE  | Pre-key stage - early development of the expected standard   |  |
| PKG  | Pre-key stage - growing development of the expected standard |  |
| HNM  | Has not met the expected standard                            | <p><b>English writing</b><br/>           HNM cannot be used for writing as there is no test.</p> <p><b>English reading, and mathematics</b><br/>           The HNM code should be used instead of the interim pre-key stage standard codes if the pupil has a test result.</p> <p><b>Science</b><br/>           The HNM code is used where pupils have not met the criteria for working at the expected standard. There are no interim pre-key stage standard codes for science.</p> |
| WTS  | Working towards the expected standard                        | To be used for English writing only.   |
| EXS  | Working at the expected standard                             | To be used for English reading, English writing, mathematics and science.  |
| GDS  | Working at a greater depth within the expected standard      | To be used for English writing only.   |

## Key stage 2: submitting teacher assessment data

May 2016

|                                    |  |
|------------------------------------|--|
| March                              | <ul style="list-style-type: none"><li>KS2 <a href="#">test administration guidance</a><sup>6</sup>, including the 'Test administrators' guide', published for 2017.</li></ul>  |
| Friday 17 March                    | <ul style="list-style-type: none"><li>Deadline for schools to complete pupil registration.</li></ul>   |
| Monday 10 April                    | <ul style="list-style-type: none"><li>All schools that have applied for early opening or compensatory marks will have been notified of the outcome of their application on NCA tools.</li></ul>  |
| Monday 24 April                    | <ul style="list-style-type: none"><li>Deadline for schools to submit applications for additional time.</li></ul>   |
| Monday 24 April to Friday 28 April | <ul style="list-style-type: none"><li>Schools receive all KS2 test materials (standard and modified versions) and stationery items.</li></ul>  |
| Monday 8 May to Thursday 11 May    | <ul style="list-style-type: none"><li>Schools administer the KS2 tests (see section 3.3).</li></ul>  |
| Thursday 11 May                    | <ul style="list-style-type: none"><li>'Access arrangements' section of NCA tools opens for special consideration applications.</li><li>The KS2 headteacher's declaration form (HDF) is available to schools on NCA tools at 5pm.</li></ul> |
| Thursday 18 May                    | <ul style="list-style-type: none"><li>Deadline for schools to apply for timetable variations on NCA tools.</li></ul>   |

|                                   |   |
|-----------------------------------|---|
| Friday 19 May                     | <ul style="list-style-type: none"> <li>Deadline for schools to submit any notifications that pupils have used a scribe, transcript, word processor or electronic or technical aid in a test.</li> <li>Deadline for schools to submit any notifications that they have administered the tests at another location or to notify STA of a pupil cheating.</li> <li>Deadline for schools to submit special consideration applications.</li> <li>Deadline for schools to submit the KS2 HDF on NCA tools.</li> <li>Schools informed by the LA on, or after, this date if they are going to receive an external moderation visit for TA.</li> </ul> |
| Monday 22 May                     | <ul style="list-style-type: none"> <li>Test materials and mark schemes available to download from <a href="#">GOV.UK</a><sup>7</sup>.</li> <li>'Teacher assessment' section of NCA tools opens.</li> </ul>  |
| Monday 5 June to Thursday 29 June | <ul style="list-style-type: none"> <li>Las undertake external moderation of KS2 English writing TA.</li> </ul>  |
| Thursday 29 June                  | <ul style="list-style-type: none"> <li>Deadline to submit TA data on NCA tools. Data submitted late will not be used in the DfE's performance tables data checking exercise.</li> </ul>   |
| Tuesday 4 July                    | <ul style="list-style-type: none"> <li>Pupil results (raw scores and scaled scores) and marked script images available on NCA tools.</li> <li>Raw score to scaled score conversion tables available on GOV.UK.</li> </ul>   |
| Friday 14 July                    | <ul style="list-style-type: none"> <li>Deadline for schools to submit review of marking applications.</li> </ul>  |
| Monday 4 September                | <ul style="list-style-type: none"> <li>Review outcomes returned to schools.</li> </ul>  |

# The scaled scores

The raw score on the tests is converted into a scaled score ensuring performance can be reported on a consistent scale for all children. Scaled scores retain the same meaning from one year to the next.

A scaled score of 100 is the expected standard. Last year, to gain a scaled score of 100 children needed to score:  
60 out of 110 in maths (55%)  
21 out of 50 in reading (42%)  
43 out of 70 in grammar, punctuation and spelling (61%)

**Steps should be taken to protect staff involved in administering the tests against allegations of maladministration.**

**Allegations of maladministration can come from misunderstandings about correct test administration.**

**To avoid this, make sure all staff, pupils and parents understand:**

- how the tests will be administered**
- the date that each test should be administered**
- what assistance is allowed in the tests**
- how any access arrangements will be used**

**<https://www.gov.uk/government/collections/national-curriculum-assessments-key-stage-2-tests>**

## 6.6 Equipment

It is important to make sure all equipment is ready and working in advance of the tests.

Pupils will require the following for one or more of the tests:

- blue/black pens, pencils, pencil sharpeners and rubbers (optional)
- rulers (showing centimetres and millimetres)
- angle measurers or protractors
- mirrors

A specific list of equipment needed for each of the tests is detailed in sections 8 to 11.

Mathematics test papers include space for working, with gridlines included where these are useful to pupils. We therefore discourage the use of additional squared paper, as pupils may lose marks if they make an error or omission when they transfer their working to the test paper. However, if using squared paper is part of normal classroom practice, then it can be used as a specific access arrangement, as described in section 6.1.

Pupils must not use glitter pens or coloured pens, in particular red. These are illegible when scanned for on-screen marking. Pupils may use highlighter pens, for example to highlight sections of the reading booklet, if this is normal classroom practice.

Every room where the tests will take place should have a clock visible to help pupils pace themselves.

# Teaching in Year 6 – part two

## Mathematics

***Jane Gill***

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**National Centre**  
for Excellence in the  
Teaching of Mathematics



**The**  
**Primary**  
**Team**



**Nottingham**  
**City Council**

# Evidence for teacher assessment: multiplicative reasoning

$19 \times 5 =$       84      95      93

Its 95 because it ends in a five or 0 when you count in fives.

$19 \times 2 =$       35      33      38

Its 38 because if counting in 2s it should be even.

$19 \times 10 =$       190      185      192

I think its 190 because when you count in tens its always ends in 0.

$28 + 38 =$       63      66      70

Its because 8+8=16

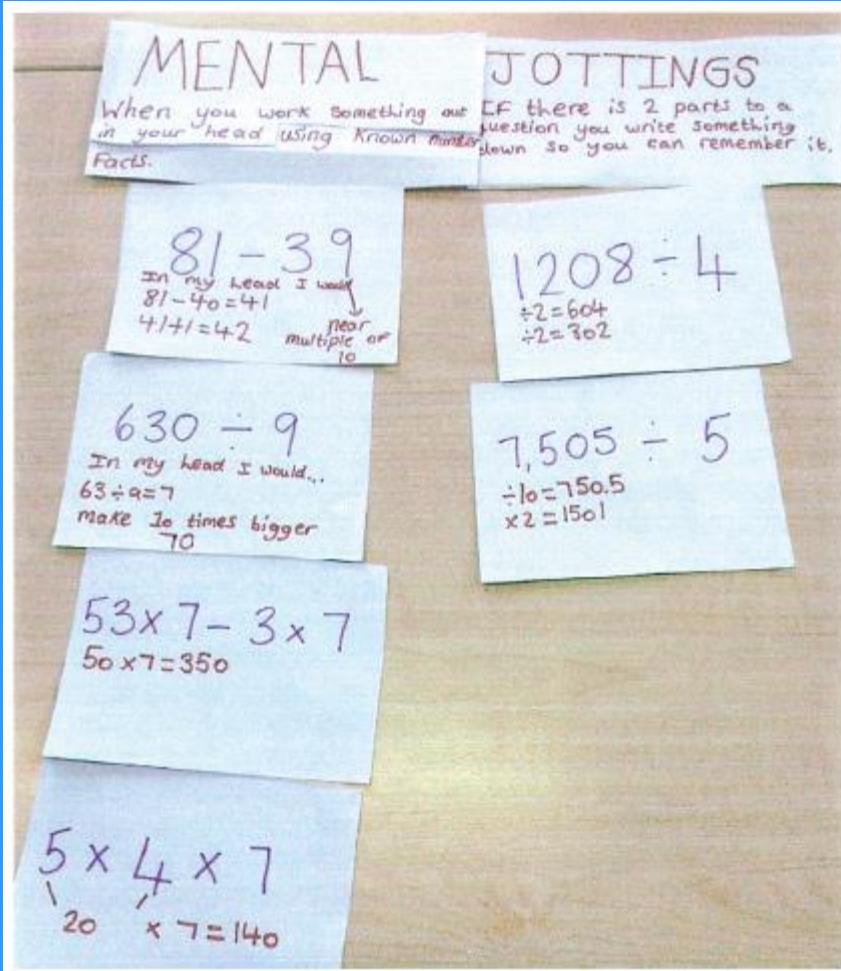
Children need opportunities to write their mathematical explanations.

## Working at the expected standard

- The pupil can demonstrate an understanding of place value, including large numbers and decimals
  - (e.g. what is the value of the '7' in 276,541?; find the difference between the largest and smallest whole numbers that can be made from using three digits;  
 $8.09 = 8 + \frac{9}{10}$   
 $28.13 = 28 + \square + 0.03$ ).
- The pupil can calculate mentally, using efficient strategies such as manipulating expressions using commutative and distributive properties to simplify the calculation (e.g.  $53 - 82 + 47 = 53 + 47 - 82 = 100 - 82 = 18$ ;  
 $20 \times 7 \times 5 = 20 \times 5 \times 7 = 100 \times 7 = 700$ ;  
 $53 \div 7 + 3 \div 7 = (53 + 3) \div 7 = 56 \div 7 = 8$ ).
- The pupil can use formal methods to solve multi-step problems
  - (e.g. find the change from £20 for three items that cost £1.24, £7.92 and £2.55; a roll of material is 6m long: how much is left when 5 pieces of 1.15m are cut from the roll?; a bottle of drink is 1.5 litres, how many cups of 175ml can be filled from the bottle, and how much drink is left?).
- The pupil can recognise the relationship between fractions, decimals and percentages and can express them as equivalent quantities
  - (e.g. one piece of cake that has been cut into 5 equal slices can be expressed as  $\frac{1}{5}$  or 0.2 or 20% of the whole cake).
- The pupil can calculate using fractions, decimals or percentages
  - (e.g. knowing that 7 divided by 21 is the same as  $\frac{7}{21}$  and that this is equal to  $\frac{1}{3}$ ;  
15% of 60;  
 $1\frac{1}{2} + \frac{3}{4}$ ;  $\frac{7}{9}$  of 108;  
 $0.8 \times 70$ ).
- The pupil can substitute values into a simple formula to solve problems
  - (e.g. perimeter of a rectangle or area of a triangle).
- The pupil can calculate with measures
  - (e.g. calculate length of a bus journey given start and end times; convert 0.05km into m and then into cm).
- The pupil can use mathematical reasoning to find missing angles
  - (e.g. the missing angle in an isosceles triangle when one of the angles is given; the missing angle in a more complex diagram using knowledge about angles at a point and vertically opposite angles).

## Teacher Assessment

To demonstrate that pupils have met the standard, teachers will need to have evidence that a pupil demonstrates consistent attainment of **all** the statements within the standard.



2016 national curriculum assessments

## Key stage 2

### 2016 teacher assessment exemplification: end of key stage 2

#### Mathematics

Working at the expected standard

January 2016



Standards & Testing Agency

- The pupil can calculate mentally, using efficient strategies such as manipulating expressions using commutative and distributive properties to simplify the calculation (e.g.  $53 - 82 + 47 = 53 + 47 - 82 = 100 - 82 = 18$ ;  $20 \times 7 \times 5 = 20 \times 5 \times 7 = 100 \times 7 = 700$ ;  $53 \div 7 + 3 \div 7 = (53 + 3) \div 7 = 56 \div 7 = 8$ ).

**Paper 1: arithmetic**

| Qu. | Content domain reference |
|-----|--------------------------|
| 1   | 3N2b                     |
| 2   | 3C2                      |
| 3   | 4C6b                     |
| 4   | 3C1                      |
| 5   | 3C2                      |
| 6   | 3C7                      |
| 7   | 5C2                      |
| 8   | 3C1                      |
| 9   | 3C7                      |
| 10  | 4C7                      |
| 11  | 3C7                      |
| 12  | 5C6a                     |
| 13  | 5C6b                     |
| 14  | 5F8                      |
| 15  | 5C7b                     |
| 16  | 5F8                      |
| 17  | 5F8                      |
| 18  | 5C2                      |
| 19  | 6C9                      |
| 20  | 6F9a                     |
| 21  | 4F8                      |
| 22  | 4C6b                     |
| 23  | 5C7a                     |
| 24  | 4F4                      |
| 25  | 6R2                      |
| 26  | 6F9b                     |
| 27  | 5F4                      |
| 28  | 6C7b                     |
| 29  | 6R2                      |
| 30  | 6C7a                     |
| 31  | 6F4                      |
| 32  | 6C7b                     |
| 33  | 6F5b                     |
| 34  | 5F5                      |
| 35  | 6F4                      |
| 36  | 6C9                      |

**Paper 2: reasoning**

| Qu. | Content domain reference |
|-----|--------------------------|
| 1a  | 3N2a                     |
| 1b  | 3N2a                     |
| 2   | 5N2                      |
| 3   | 3C2                      |
| 4a  | 4S1                      |
| 4b  | 5S1                      |
| 5   | 5C5c                     |
| 6   | 4G2c                     |
| 7a  | 6F2                      |
| 7b  | 6F2                      |
| 8   | 5F10                     |
| 9   | 3M9a                     |
| 10  | 3F2                      |
| 11  | 5M9c                     |
| 12a | 6A2                      |
| 12b | 6A2                      |
| 13  | 6R1                      |
| 14  | 6C5                      |
| 15  | 5M5                      |
| 16a | 6N2                      |
| 16b | 6N2                      |
| 17a | 6G4b                     |
| 17b | 6G4a                     |
| 18  | 6C8                      |
| 19  | 6C8                      |
| 20  | 6P2                      |

**Paper 3: reasoning**

| Qu. | Content domain reference |
|-----|--------------------------|
| 1   | 3C1                      |
| 2a  | 6N5                      |
| 2b  | 6N5                      |
| 3   | 4M4b                     |
| 4a  | 6A2                      |
| 4b  | 6A2                      |
| 5   | 5F8                      |
| 6   | 4F10b                    |
| 7a  | 4G4                      |
| 7b  | 4G4                      |
| 8   | 6C8                      |
| 9a  | 5S1                      |
| 9b  | 5S1                      |
| 10  | 5M8                      |
| 11  | 6C7a                     |
| 12  | 4P2                      |
| 13  | 5F10                     |
| 14a | 6M5                      |
| 14b | 6M5                      |
| 15  | 5N4                      |
| 16  | 6R4                      |
| 17  | 6M7b                     |
| 18  | 6G2a                     |
| 19  | 6N6                      |
| 20  | 5F10                     |
| 21  | 6C8                      |

# 2016 content domain coverage

Last year 50 questions were from Y3, 4 & 5 (which amounted to 62 marks).

60 marks were needed for a scaled score of 100: the expected standard.

The tests are generally presented in order of difficulty, but there will always be some later questions that children will be able to do.

# Paper 1: arithmetic (36 questions, 40 marks)

24

$$\frac{4}{7} + \frac{5}{7} =$$

92% of children got this right.

25

$$20\% \text{ of } 1,800 =$$

74% got this right.

29

$$15\% \times 440 =$$

But only 36% got this right!

34

$$\frac{2}{5} \times 140 =$$

And only 27% got this right.

## Teaching tips:

Make sure children read the x sign as 'of'.

*15% of 440 is quite easy to do mentally.*

Ensure children can connect key fractions to their decimal and percentage equivalents.

$$\text{e.g. } \frac{1}{5} = 20\% = 0.2 = \div 5$$

*Two fifths of 140... divide by 5 then double the answer.*

# New content but not too difficult:

33

$$\frac{3}{5} \div 3 =$$



36

$$60 - 42 \div 6 =$$



Treat the **fifths** as the ‘noun’ or the ‘thing’...

3 ‘things’ shared between 3...

How many of the ‘thing’ do they each get?

Remember **BODMAS** for question 36 and then the calculation is quite straightforward.

$$7 + 3 \times 5 =$$



Encourage children to explain why the answer to this is 22 and not 50.

# Ensure your year 6 children have had experience of:

$459 \times 0 =$

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$10\,000\,000 - 101 =$

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|  |  |  |  |  |  |  |

$$\boxed{\phantom{0000}} = 6853 - 684$$

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$1\,000 \times 30.7 =$

|  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

Write the number that is 300,000 less 8 million

Gary took part in a quiz show and won a million pounds.

He spent £20 000 on a holiday.

Then he spent half of the money left on a house.

How much did Gary's house cost?

# Spend time helping children to memorise useful key facts

**Fractions-decimals-percentages  
(on a number line)**

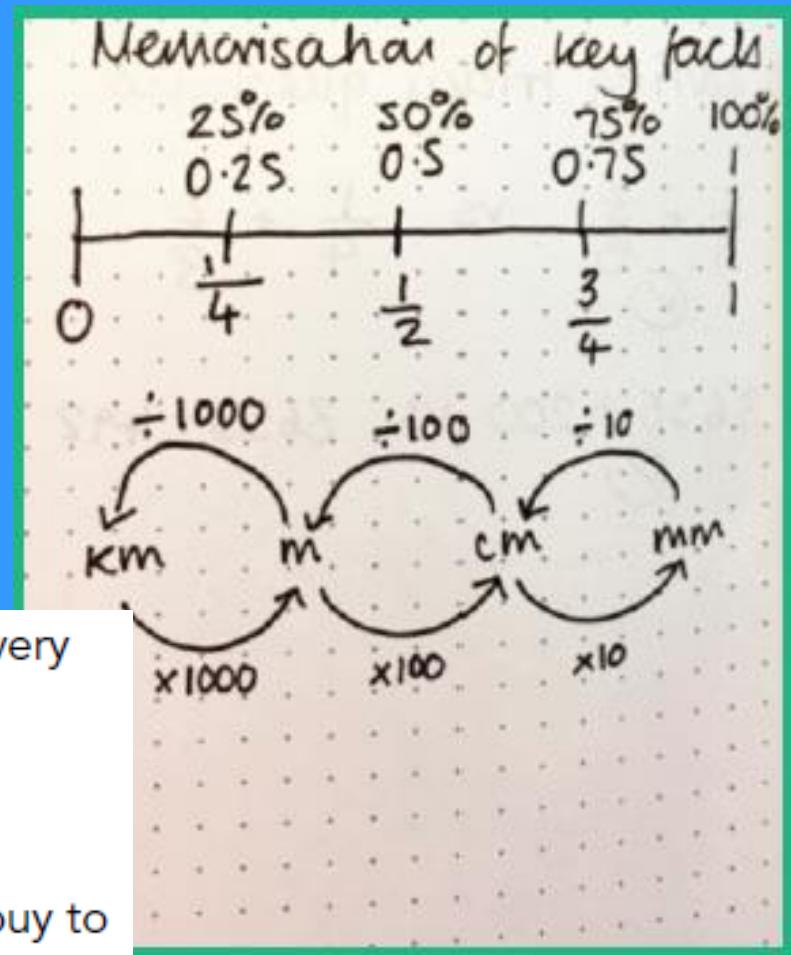
**Different unit conversions**

On average a lion in a zoo eats 4950g of meat every day.

The meat comes in 6kg packs.

How many packs of meat does the zoo need to buy to feed the lion for one week?

Show your method.



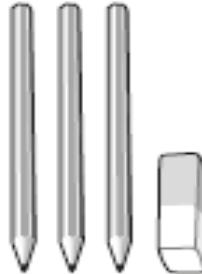
# Papers 2 and 3: reasoning

## Finding a starting point in a problem

6 pencils cost £1.68



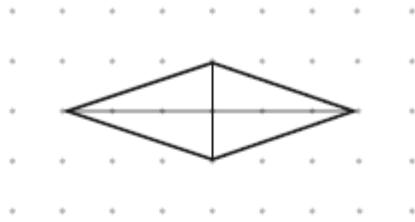
3 pencils and 1 rubber cost £1.09



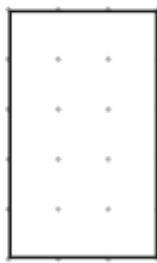
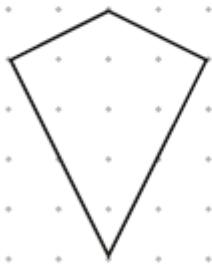
What is the cost of 1 rubber?

Children can use the pictures to help them understand the problem. Encourage them to annotate the picture or to draw a new model.

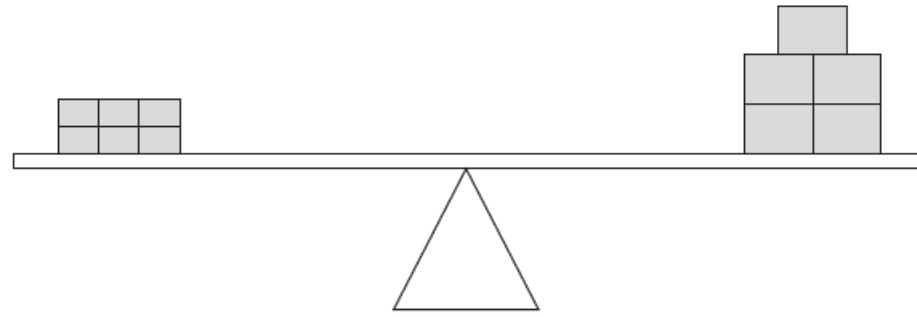
The diagonals of this quadrilateral cross at right angles.



Tick **all** the quadrilaterals that have diagonals which cross at right angles.



6 small bricks have the same mass as 5 large bricks.



The mass of one small brick is 2.5 kg.

What is the mass of one large brick?

**Annotating the drawings  
will help children to see  
solutions.**

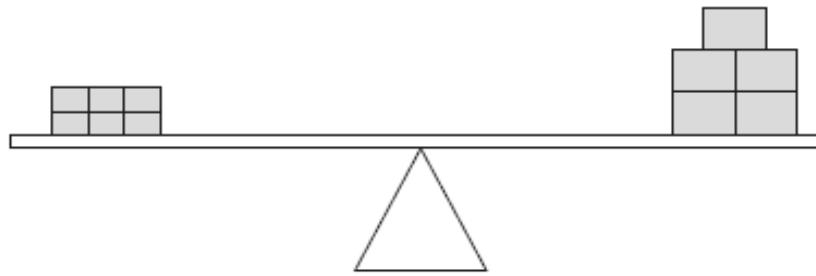
# Use reasoning prompts to help children articulate what they can see and work out.

**The thing I noticed was .....**

**I already knew ..... so this helped me work out .....**

16

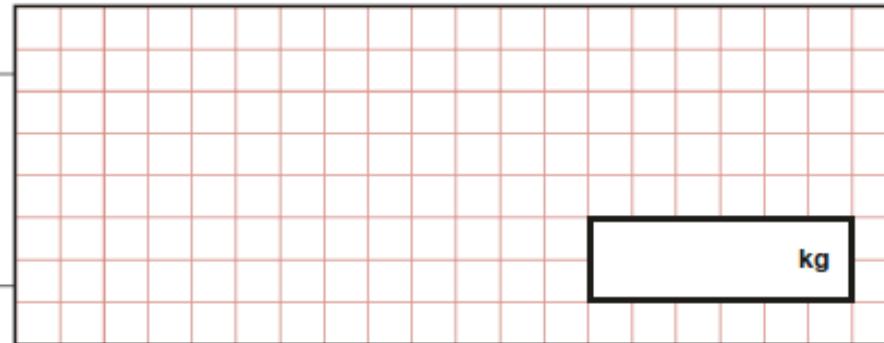
6 small bricks have the same mass as 5 large bricks.



The mass of one small brick is 2.5 kg.

What is the mass of one large brick?

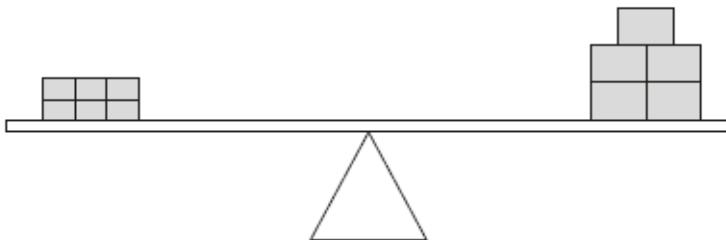
Show your method



2 marks

**I know this is true because .....**

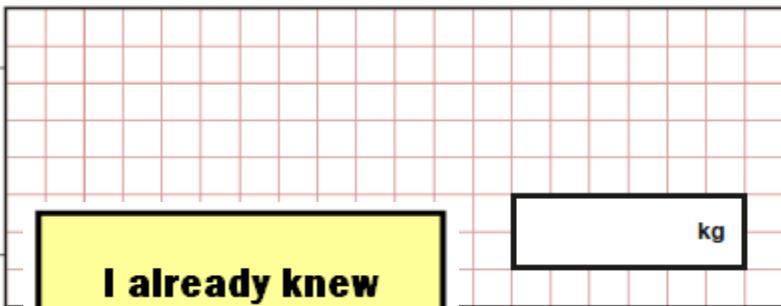
6 small bricks have the same mass as 5 large bricks.



The mass of one small brick is 2.5 kg.

What is the mass of one large brick?

Show  
your  
method



## KS2 SATs 2016 paper 3: reasoning



I already knew that the mass of 2 small bricks would be 5kg so this helped me work out the mass of 6 small bricks.  
6 small bricks weigh 15kg and 5 large bricks weigh 15kg.  
I know this is true because the scales balance.

The thing I noticed was .....

When I saw this it made me think about .....

I know this is true because .....

I realised this couldn't be right because .....

When I got stuck I decided to try .....

The connection I think is important is .....

The thing that helped me see the connection was .....

I thought the answer looked right because .....

The way I would describe the pattern is .....

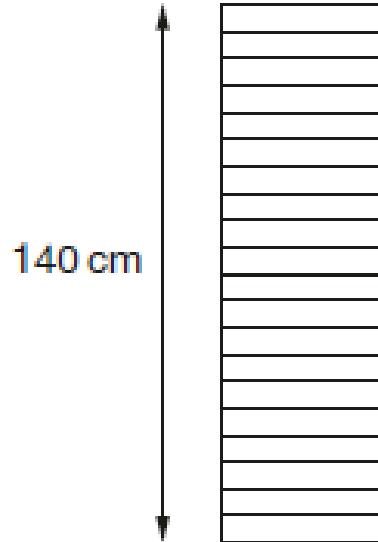
I wondered what would happen if .....

I already knew ..... so this helped me work out .....

The strategy I used was ..... I chose this strategy because .....

# Using the drawings

A stack of 20 identical boxes is 140 cm tall.



Stefan takes three boxes off the top.

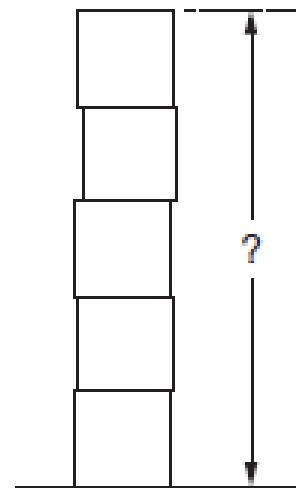
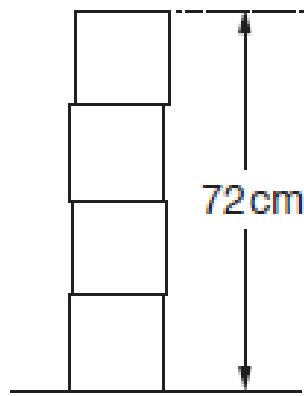
How tall is the stack now?

What do you know?  
What can you find out?

Lisa has some boxes that are all cubes of the same size.

She uses four of the boxes to make a pile with a height of 72cm.

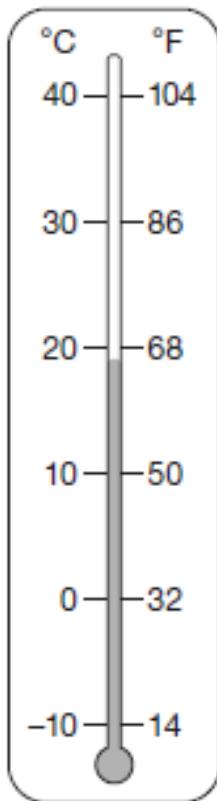
She puts one more box on top of the pile.



Work out the height of the pile of five boxes.

# Using the drawings

This thermometer shows temperatures in both  $^{\circ}\text{C}$  and  $^{\circ}\text{F}$ .



Work out what  $25^{\circ}\text{C}$  is in  $^{\circ}\text{F}$ .

What do you know?

What can you find out?

Look for easy ways.

A shape is made up of a square and rectangle.



The perimeter of the shape is 70cm.

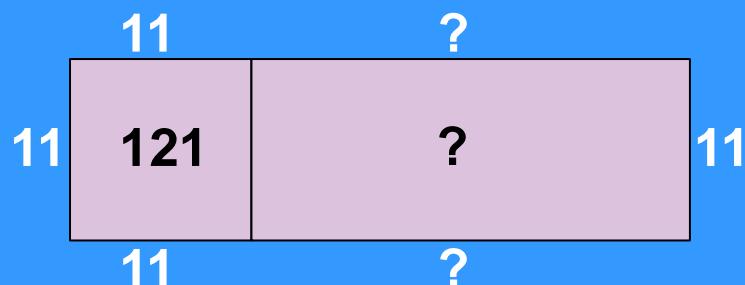
The area of the square is 12cm<sup>2</sup>

What is the area of the rectangle?

What do you know?

Label what you know.

What can you find out?



What can you find out?

$$70 - 44 = 26$$

$$26 \div 2 = 13$$

$$13 \times 11 = 143$$

A shape is made up of a square and rectangle.



The perimeter of the shape is 70cm.

The area of the square is  $121\text{cm}^2$

What is the area of the rectangle?

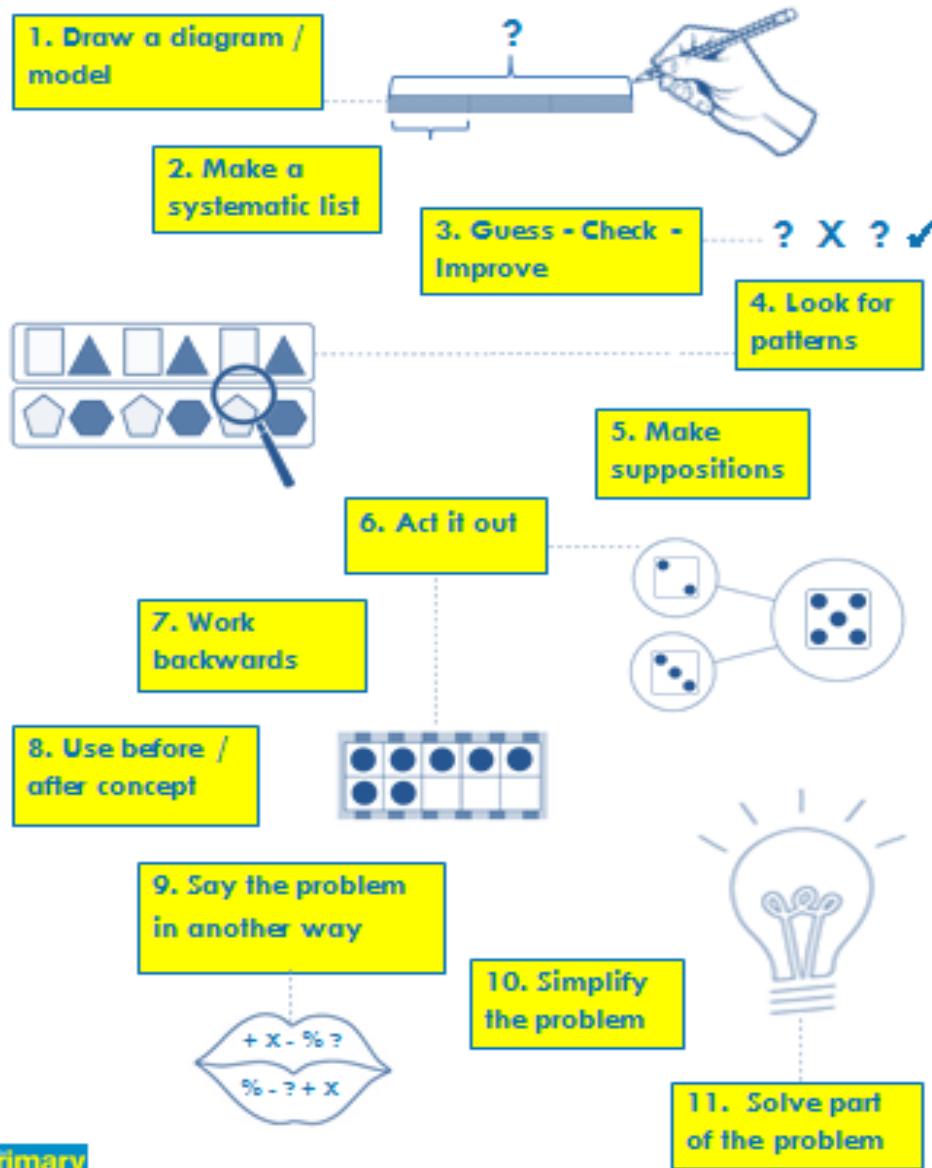
# Teaching children heuristics to help them solve mathematical problems

The key to successful problem solving is finding the starting point, which requires flexible and logical thinking, not simply following a procedure.

The use of **heuristics** (problem-solving rules) can help greatly to improve a child's problem solving performance.

The word **heuristics** originates from the Latin word *heuristicus*, which means to find out or to discover.

# 11 Heuristics for Solving Problems



It is extensively recognised and accepted that heuristics should be and can be taught. The crucial point is that heuristics are about ideas, not routine steps.

Children will learn to choose the most suitable heuristics resulting in more efficient ways of solving a problem.

Henry makes a 3-digit number.



My number lies  
between 209 and 220



The digits sum to 9

What number did Henry make?

What heuristics could children use to help them solve this problem?

# Guess and check

# Make a supposition

# Make a list

# Look for patterns

Henry makes a 3-digit number.



My number lies  
between 209 and 220



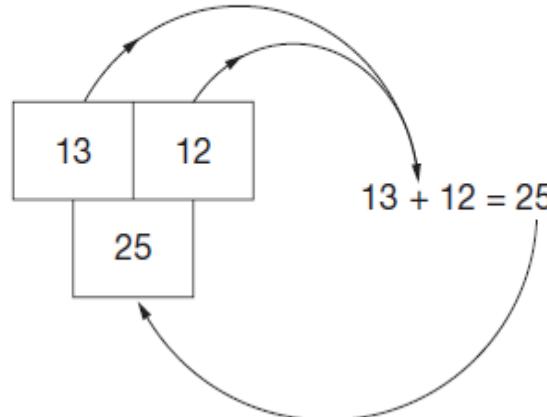
The digits sum to 9

What number did Henry make?

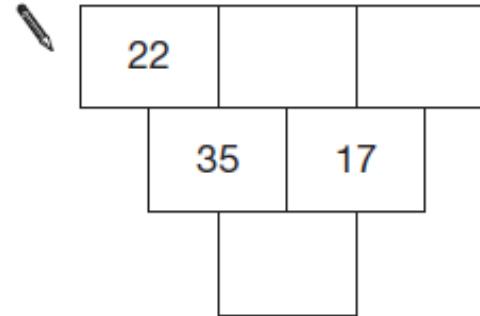
# What heuristics could children use for this?

In these number grids, two numbers are added to give the number below.

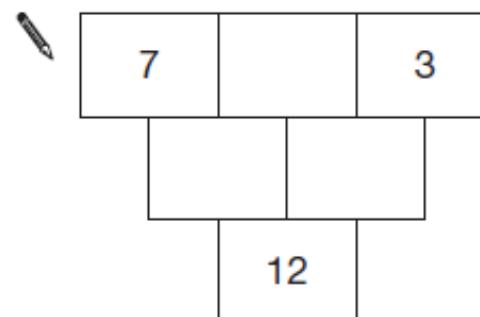
Example:



Write numbers in the number grids below to make them correct.



A diagram showing a number grid with three boxes. The top-left box contains the number 22. The bottom-left box contains the number 35, and the bottom-right box contains the number 17. A pen icon is positioned to the left of the grid, indicating where to write.



A diagram showing a number grid with three boxes. The top-left box contains the number 7, and the top-right box contains the number 3. The bottom box contains the number 12. A pen icon is positioned to the left of the grid, indicating where to write.

# Heuristic: guess - check – improve

Put the **same** number in each box to make the calculation correct.

$$140 + 10 - \boxed{\phantom{000}} = \boxed{\phantom{000}} - 30$$

Consider how children could use guess-check-improve to solve this problem...

$$140 + 10 - 100 = 50$$

$$100 - 30 = 70$$

$$140 + 10 - 110 = 40$$

$$110 - 30 = 80$$

$$140 + 10 - 90 = 60$$

$$90 - 30 = 60$$

**Guess and check** (or trial and improvement) is perhaps an undervalued skill as children may feel they are only using it because they do not know the 'right' way to solve the problem.

Show children that trying something out will always give more insight into the context which will give the solver a better idea of what to try next. Trial and improvement is often the start of working systematically.

Find the two **square numbers** that add together to make 100.

\_\_\_\_\_ and \_\_\_\_\_

# Heuristic: guess and check

Use 4 **different** digits to complete this multiplication calculation:

$$\boxed{\phantom{00}} \times \boxed{\phantom{00}} \times \boxed{\phantom{00}} \times \boxed{\phantom{00}} = 168$$

How many **different solutions** can you find?  
Have you found them all? Explain your reasoning.

Support children to think logically in order to make a calculated guess...

Why can't 5 be one of the digits?

Why can't 9 be one of the digits?

# Heuristic: guess and check

Use 4 **different** digits to complete this multiplication calculation:

$$\boxed{\phantom{0}} \times \boxed{\phantom{0}} \times \boxed{\phantom{0}} \times \boxed{\phantom{0}} = 168$$

$$\del{6 \times 4 \times 3 \times 2 = 144}$$

$$7 \times 6 \times 4 \times 1 = 168$$

$$3 \times 7 \times 8 \times 1 = 168$$

$$3 \times 7 \times 4 \times 2 = 168$$

What do you notice?

# Which number where?

$$\begin{array}{ccccc} \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \hline = & & = & & = \end{array} \quad = \quad \begin{array}{c} \boxed{40} \\ \boxed{144} \\ \boxed{63} \end{array}$$

112    108    30

$$\begin{array}{ccccc} \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \hline = & & = & & = \end{array} \quad = \quad \begin{array}{c} \boxed{210} \\ \boxed{24} \\ \boxed{72} \end{array}$$

84    45    96

$$\begin{array}{ccccc} \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \hline = & & = & & = \end{array} \quad = \quad \begin{array}{c} \boxed{108} \\ \boxed{42} \\ \boxed{80} \end{array}$$

192    15    126

$$\begin{array}{ccccc} \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} & \times & \boxed{\phantom{00}} \\ \hline = & & = & & = \end{array} \quad = \quad \begin{array}{c} \boxed{35} \\ \boxed{24} \\ \boxed{432} \end{array}$$

120    18    168

# Divisibility Rules



**Dividing by 1:** If you divide any whole number by 1, you always get a whole number.



**Dividing by 6:** If the number is divisible by both 3 and 2, it is divisible by 6 as well. For example, 2157924 is divisible by 6 because it is even (divisible by 2) and the digits add up to 30, which is divisible by 3.



**Dividing by 2:** Even numbers "evenly" divide into 2. Odd numbers divide into 2 with an "odd one out".



**Dividing by 7:** To find out if a number is divisible by 7, take the last digit, double it, and subtract it from the rest of the number without the last digit. If you get an answer divisible by 7 (including 0), then the original number is divisible by 7. If you don't know the new number's divisibility, you can apply the rule again. For example, 161 is divisible by 7 because  $2 \times 1$  (the last digit) = 2 and  $16 - 2 = 14$ , which is divisible by 7.



**Dividing by 3:** Add up the digits (twice, if necessary); if the sum is divisible by 3, then the number is too. Let's say you need to divide 123:  $1 + 2 + 3 = 6$ , which is divisible by 3, so 123 is divisible by 3. Another example: 678678. Add  $6 + 7 + 8 + 6 + 7 + 8 = 42$ ;  $4 + 2 = 6$ , which is divisible by 3. That means 678678 is divisible by 3.



**Dividing by 4:** Look at the last two digits. If they are divisible by 4, the number is as well. For example, the last two digits of 2357924 are 24, which is divisible by 4. Therefore, 2357924 is divisible by 4.



**Dividing by 5:** If the last digit is a 5 or a 0, then the number is divisible by 5. For example, 2357925 is divisible by 5, because the last digit is a 5.



**Dividing by 9:** Add the digits. If they are divisible by 9, then the number is as well. For example 52866 is divisible by 9 because  $5 + 2 + 8 + 6 + 6 = 27$ , and 27 is divisible by 9.



**Dividing by 10:** If the number ends in 0, it is divisible by 10.



**Dividing by 11:** Keep subtracting the last digit from the previous digits until you can tell if the resulting number is divisible by 11. For example: 645634 is divisible by 11 because  $64563 - 4 = 64559$ ;  $6455 - 9 = 6446$ ;  $644 - 6 = 638$ ,  $63 - 8 = 55$ , and 55 is divisible by 11.

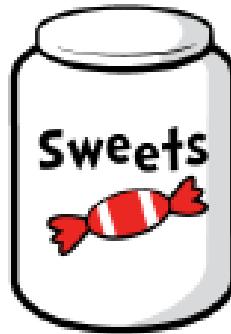


**Dividing by 12:** Check for divisibility by 3 and 4.

**Dividing by 8:** If the last three digits of a number are divisible by 8, then so is the whole number. How do you check the last three digits? If the first digit is even, and the last two digits are divisible by 8, the number is divisible by 8. If the first digit is odd, subtract 4 from the last two digits; the number will be divisible by 8 if the resulting last two digits are. For example:

- 2448: Check the last three digits, 448. Here, 4 is even and 48 is divisible by 8, so 2448 is also divisible by 8.
- 192: Here, 1 is odd, so you need to subtract 4 from the last two digits:  $92 - 4 = 88$ ; 88 is divisible by 8, so 192 is as well.

A jar contains 30 sweets.



The weight of the jar and sweets is 620g.

David eats 12 sweets.

The weight of the jar and sweets is now 440g.

How much does the jar weigh?

What heuristics could children use to help them solve this problem?

**Solve part of the problem**

**Work backwards**

**Draw a model**

**Before-after concept**

A jar contains 30 sweets.



The weight of the jar and sweets is 620g.

David eats 12 sweets.

The weight of the jar and sweets is now 440g.

How much does the jar weigh?

# Heuristic: draw a model

# Algebra

$$2q + 4 = 100$$

Work out the value of  $q$ .

$$q =$$

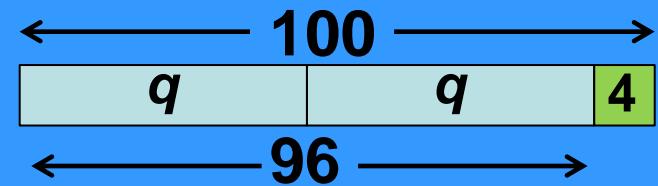
$$2q + 4 = 100$$

Work out the value of  $q$ .

$q =$

#### Word Problem Checklist

1. What do you know and what don't you know?
2. What is the whole and what are the parts?
3. Draw a model and label what you know.
4. Write the calculation.
5. Estimate then do the calculation.
6. Check the problem and your answer.



Encourage children to use a bar model to visualise algebra questions

# Heuristic: draw a model

# Algebra

One egg and one slice of toast costs £1.94



Three eggs and two slices of toast costs £5



How much does one slice of toast cost?

A football and toy train together weigh 360g.



Three footballs and two toy trains weigh 810g.



Find the weight of a toy train.

## Heuristic: draw a model

## Comparison

A florist stocks three types of roses; pink, white and red.

Half of all the roses are pink.

There are 28 red roses and there are twice as many white roses as red ones.

The florist sells all of the roses at 20p each.

How much money does she receive for the roses?

## Heuristic: draw a model

## Ratio

There are three 9 year olds to every five 10 year olds in Year 4. There are 88 children in Year 4.

How many are 10 years old?

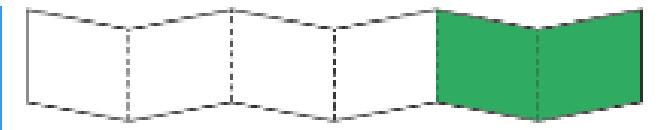
# Heuristic: draw a model

# Fractions

A tablet computer was reduced by  $1/3$  in a sale to £108. What was the original price?



**Joe spent two thirds of his money on a game which cost £36. He then spent half of his remaining money on a book. How much money did he have left?**



# Heuristic: before-after concept

Mel, Martina and Pat have the **same** number of cards.



Mel and Martina each give Pat 4 cards.

Pat now has 17 cards.

How many cards do they have altogether?

Draw the 'before' model.

Then draw the 'after' model.

Label what you know.

# Heuristic: before-after concept

Toby and Ada each have a pot containing the same number of counters.



Ada puts half of her counters into Toby's pot.

Toby wants to make the number of counters in each pot equal again.

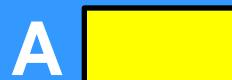
What fraction of the counters in his pot must Toby put into Ada's pot?

# Heuristic: before-after concept

Before:



After:



# Heuristic: work backwards

13

Lara chooses a number less than 20

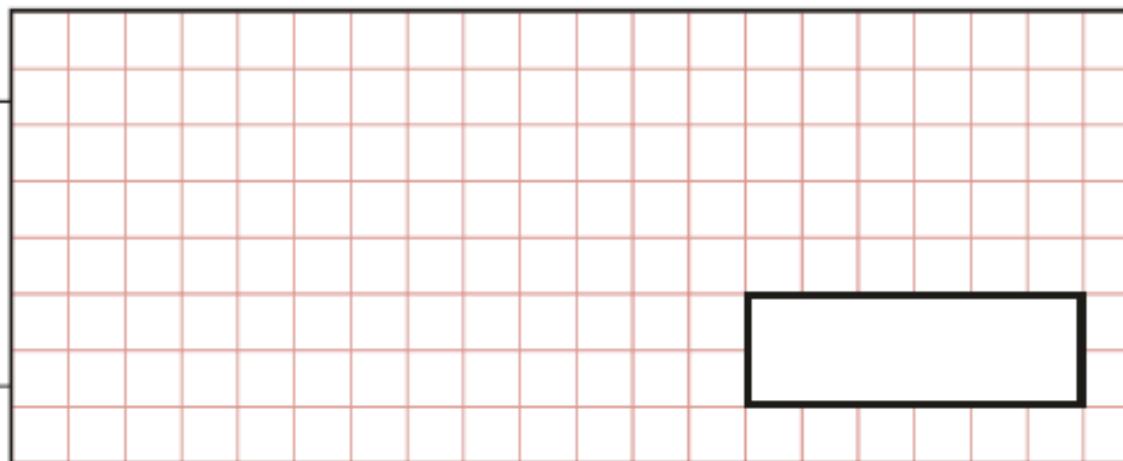
She divides it by 2 and then adds 6

She then divides this result by 3

Her answer is 4.5

What was the number she started with?

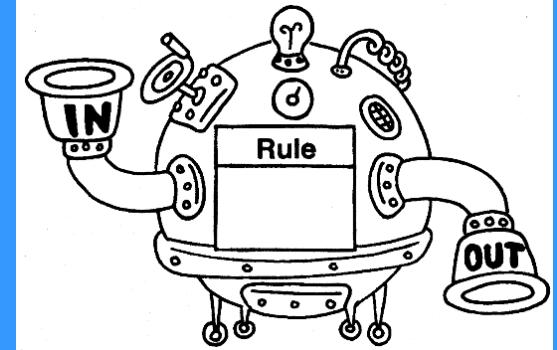
Show  
your  
method



A 10x10 grid for working out the answer. A large rectangular box is overlaid on the grid, with a smaller rectangular box inside it for writing the final answer.

2 marks

# Heuristic: work backwards



## Function machines / mystery numbers...

If you add 5 to the number the answer is 36.

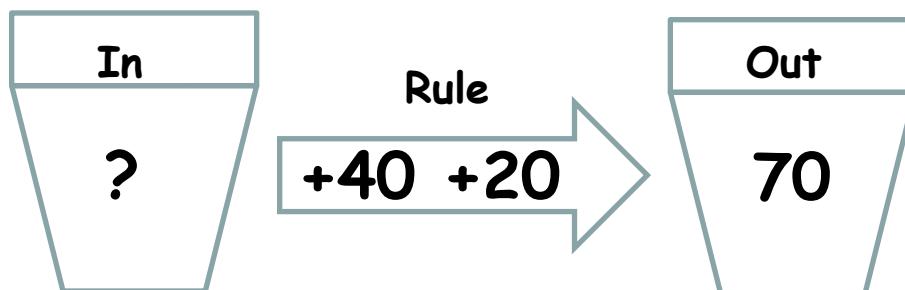
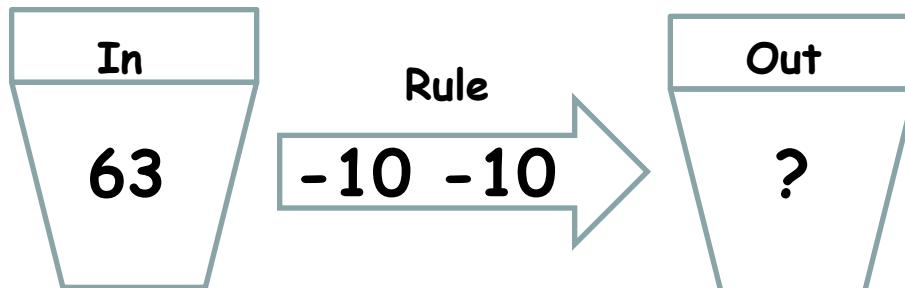
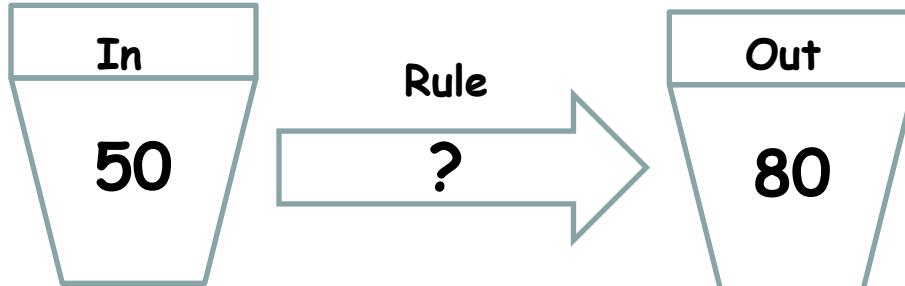
Halve the number then add 6 and the answer is 20.

Add 7 then double it, then subtract 2, and the answer is 22.

Multiply by 5, subtract 3 & divide by 8, and the answer is 4.

Choose any number, multiply it by 2, then multiply by 10, then divide by 100 and then multiply by 5.

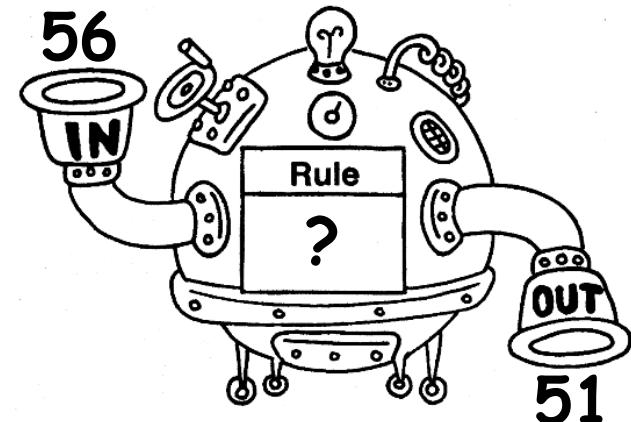
What answer do you get? Why?



$$10 + 40 + 20 = 70$$

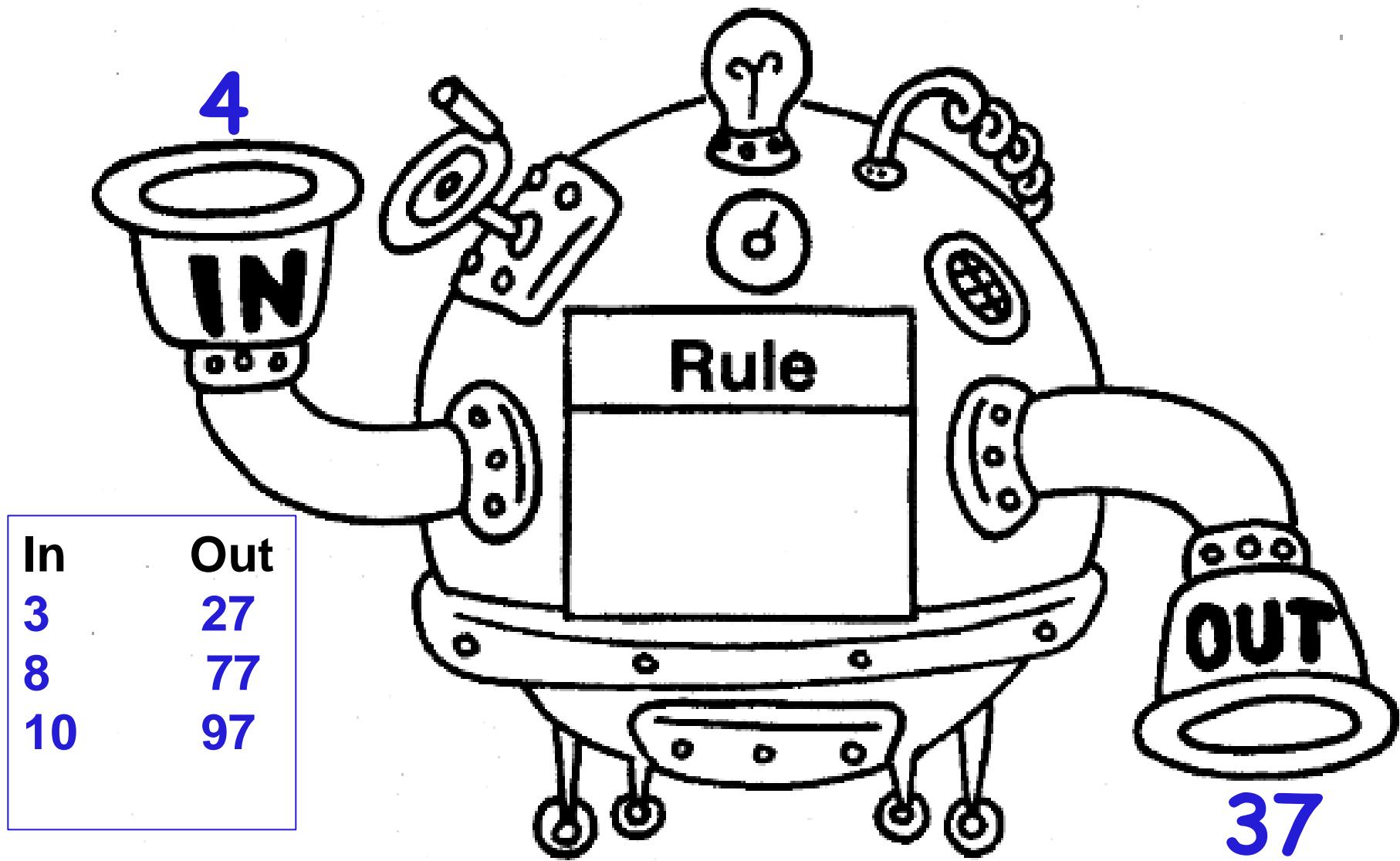
and

$$70 - 20 - 40 = 10$$

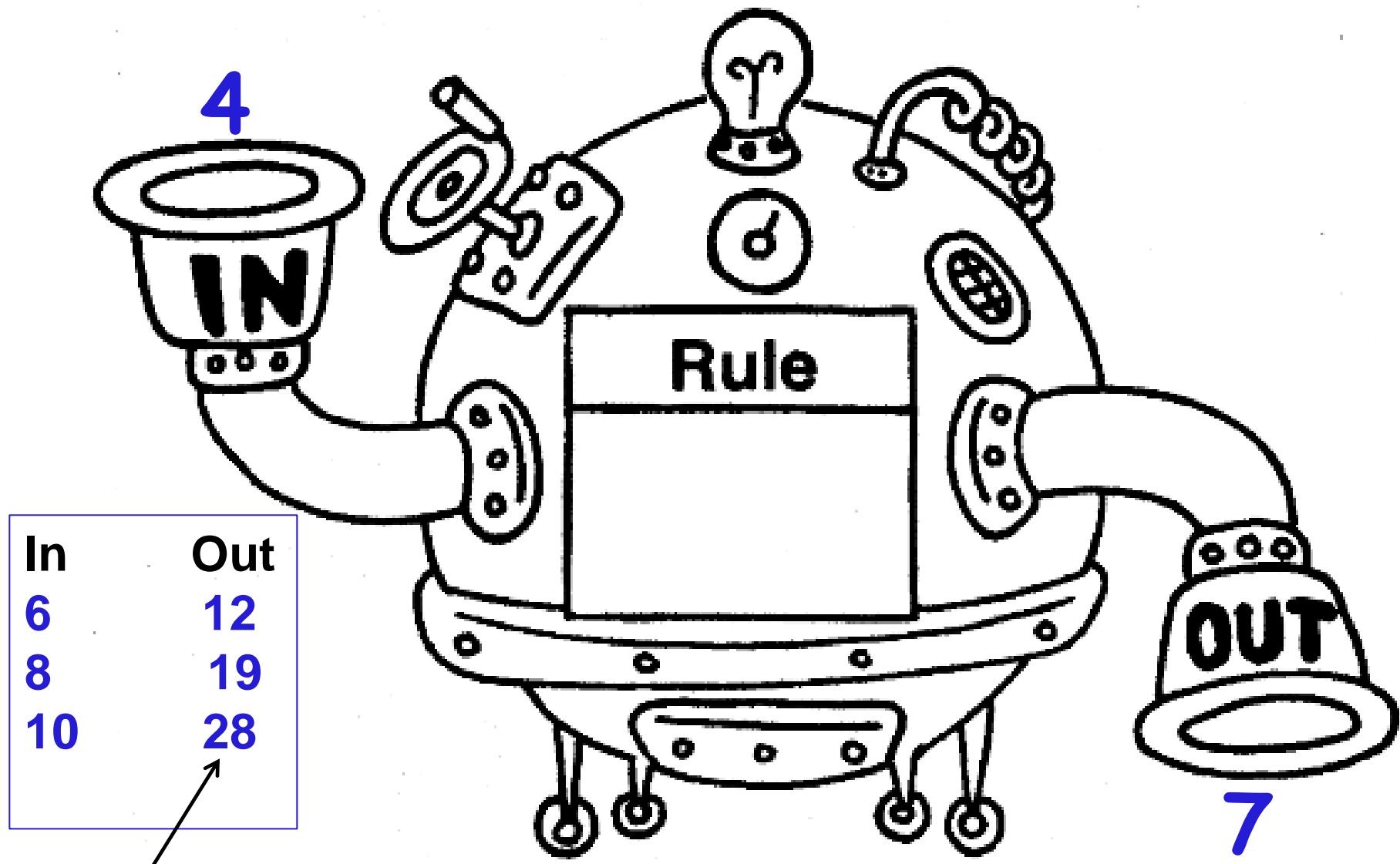


**Encourage children to create and use function machines and to write the related equations.**

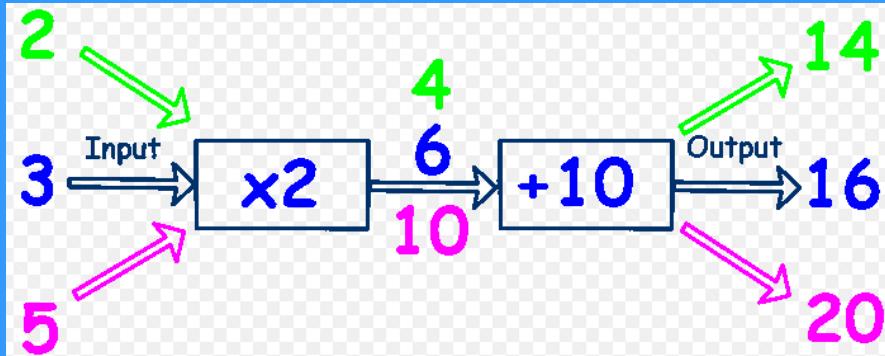
# Guess my rule (2 steps or 3 steps)



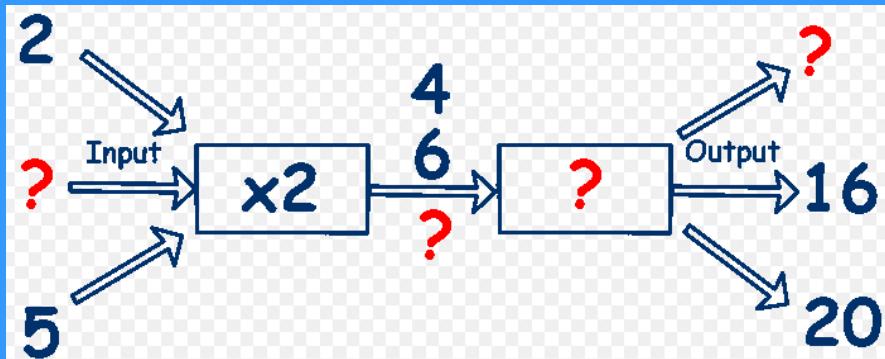
# Guess my rule (2 steps or 3 steps)



What do you notice about these numbers?



Use function machines to help children with missing number problems:



What is my rule?

Rule is  $x3$  then +1

| In | Out |  | In | Out |
|----|-----|--|----|-----|
| 6  | 11  |  | 2  | 7   |
| 10 | 19  |  | 6  |     |
| 12 | 23  |  | 11 |     |
|    |     |  | 5  |     |
|    |     |  |    |     |
|    |     |  |    |     |
|    |     |  |    |     |

# Heuristic: act it out

David and Anna have these cards.

1

2

3

5

6

7

David uses four of the cards to make a **pair of equivalent fractions**.

Write numbers in the boxes to show how David can do this.



$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

Anna has the same cards.

She uses four of the cards to make a **different pair of equivalent fractions**.

Write numbers in the boxes to show how Anna can do this.

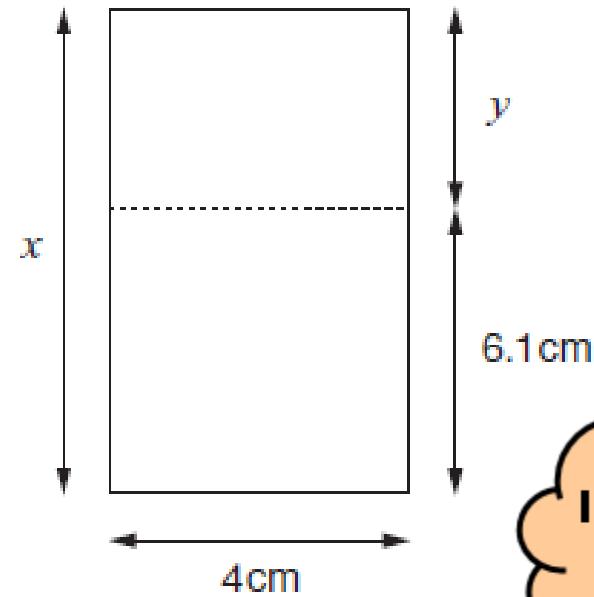


$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

# Heuristic: solve part of the problem

Look at the rectangle.

I already knew  
..... so this  
helped me work  
out .....



Not drawn  
accurately

I know this is true  
because .....

The thing  
I noticed  
was .....

What do you  
know?

The total area of the rectangle is  $40\text{cm}^2$

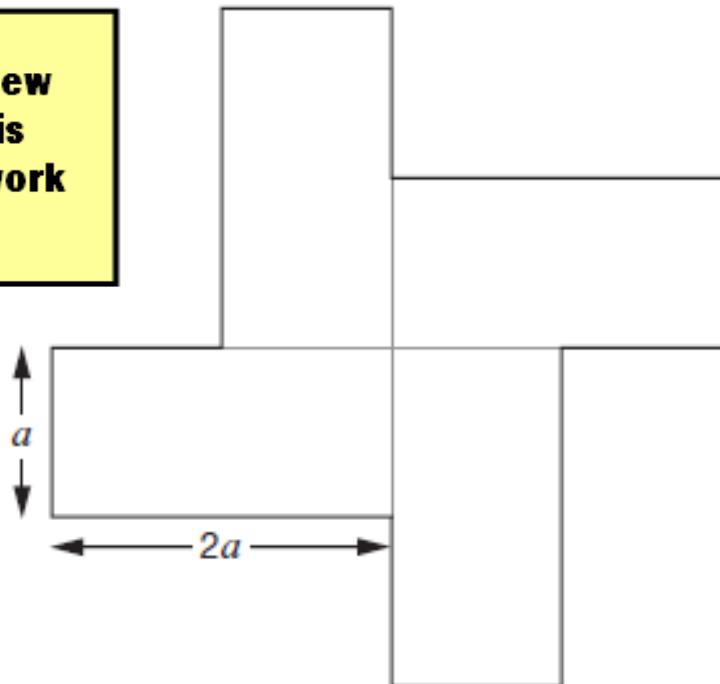
Work out lengths  $x$  and  $y$

# Heuristic: solve part of the problem

This shape is made of four congruent rectangles.

Each rectangle has side lengths  $2a$  and  $a$

I already knew  
..... so this  
helped me work  
out .....



Not drawn  
accurately

The perimeter of the shape is 80cm.

Work out the area of the shape.

The thing  
I noticed  
was .....

What do you  
know?

I know this is true  
because .....

# Heuristics: say the problem in another way / simplify the problem

18

Write the missing number.

$$70 \div \boxed{\phantom{00}} = 3.5$$

**Say the problem in another way**

70 divided by  $3\frac{1}{2}$  = ? **or** 3.5 times something is 70 **or**

How many  $3\frac{1}{2}$ s in 70?

**Simplify the problem**

Double each number in the problem ( $140 \div \boxed{\phantom{00}} = 7$ )

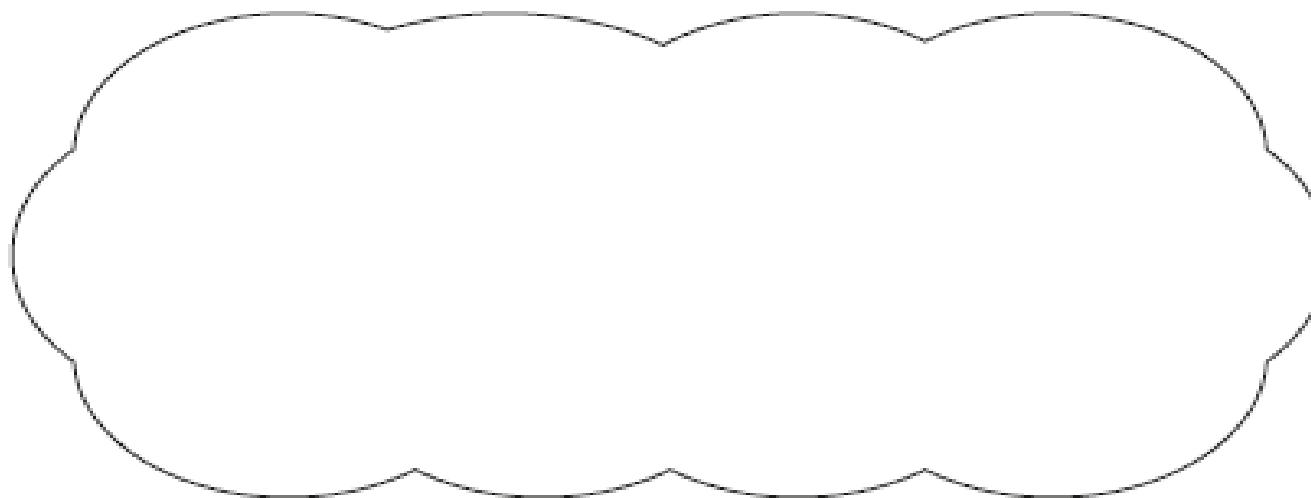
Half of 70 is 35 and  $35 \div 3.5 = 10$  so  $70 \div 3.5$  must be double that (20)

If  $70 \div 10 = 7$ ,  $70 \div 20 = 3.5$

21

$$5,542 \div 17 = 326$$

Explain how you can use this fact to find the answer to  $18 \times 326$



1 mark

**Say the problem in another way.  
Explain in a clear, mathematically correct manner.**

21

An explanation that shows that 5,868 can be made by adding 326 to  $17 \times 326$ , e.g.

- '5542 + 326 = 18 × 326'
- '18 × 326 is 326 more than 5,542'
- 'Because this is the same as  $17 \times 326 = 5542$  so add one more 326 to get the answer'
- 'You add 326 to 5,542 and your answer will be correct'
- 'Because you can add 326 to the answer of  $17 \times 326$ '
- '5542 + 326'.

1m

Do not accept an explanation that simply calculates  $326 \times 18 = 5,868$

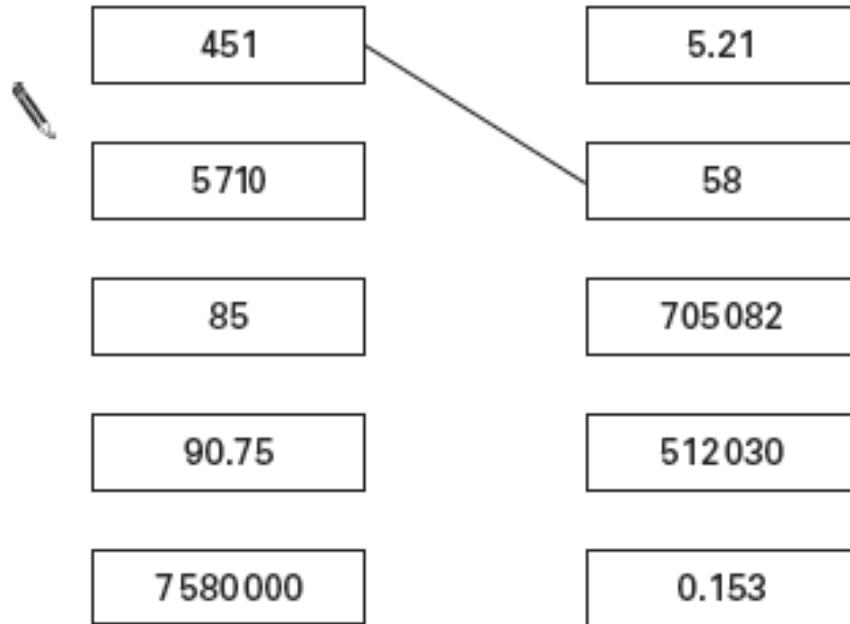
Do not accept vague or incomplete, or incorrect explanations, e.g.

- 'You could add another 326'
- 'The difference between 17 and 18 is 1 so you add 326 and that is one more'
- 'Because if you turn the question around you would see that  $17 \times 326 = 5542$  so all you need to do is times the number one more time'
- '5,542 + 326 because it is one more'.
- $5868 - 326 = 5542$

# Question styles in the KS2 Mathematics test

Draw lines to match the numbers where the digit 5 represents the same value.

The first is done for you.



$$7 < \dots \dots \dots 10 \dots \dots \dots < 11$$

$$7 < \dots \dots \dots < 8.5$$

$$7 < \dots \dots \dots < 7.5$$

$$7 < \dots \dots \dots < 7.05$$

**Completing statements / equations**

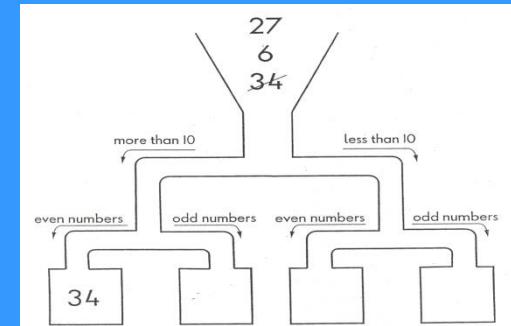
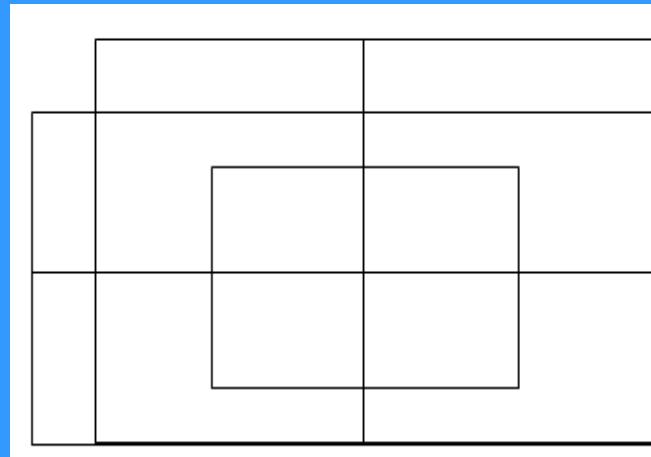
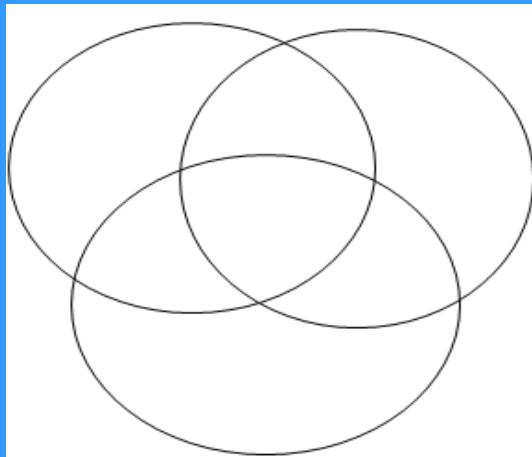
Matching/sorting in a variety of ways

# Using sorting activities to help children revise number properties and make connections

Create your own sorting diagram for multiples of 3 & multiples of 8 that are less than 50.

# Three attribute sorting diagrams

You can use John Venn's diagram, Lewis Carroll's square with a subset, or you can design your own (and name it after yourself!)



# Compare your sorting diagrams.

What do you notice?

What's the same? What's different?

What generalisations can you make?

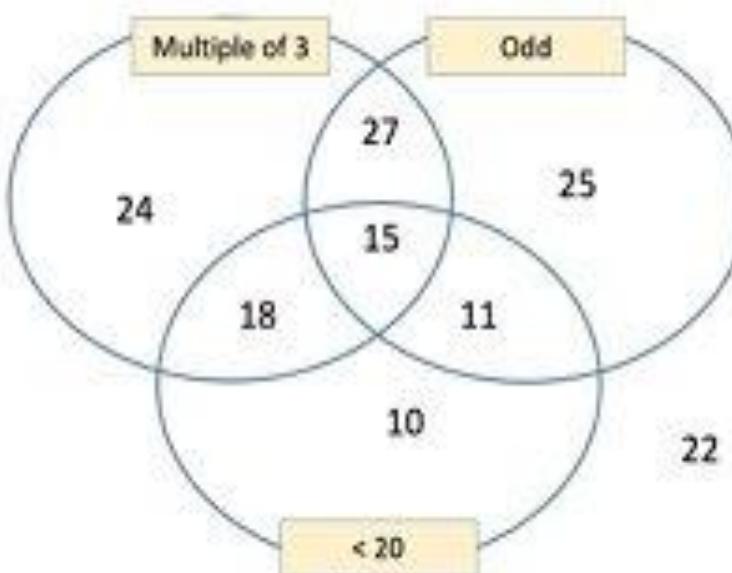
Use your diagram to answer this SAT question from last year:

Write all the common multiples of 3 and 8 that are less than 50

*2016 KS2 SATs Paper 2 Question 14*

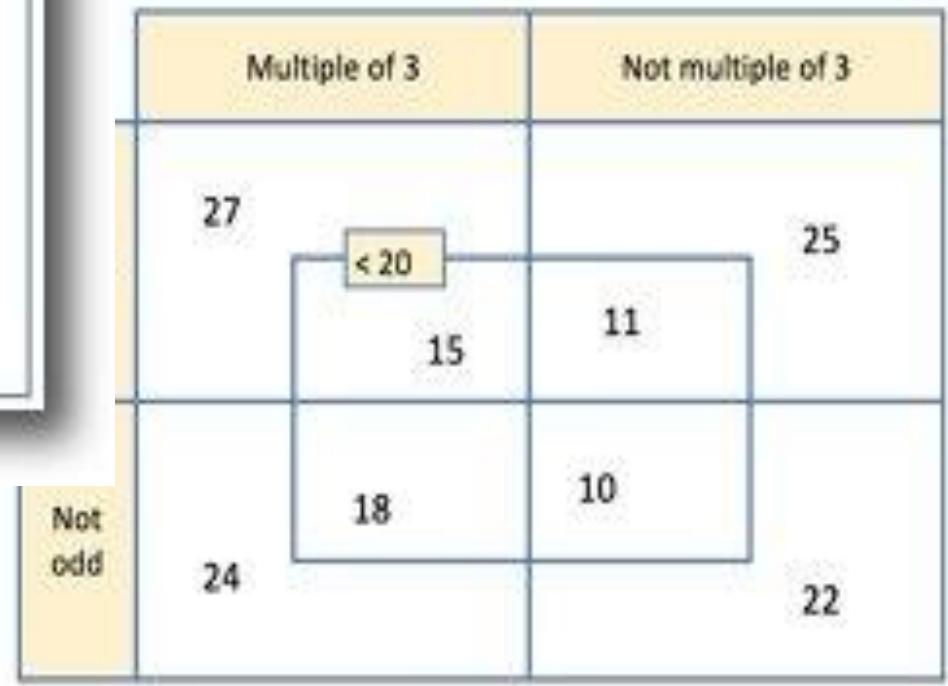
# Three attribute sorting diagrams

Encourage children to compare Venn and Carroll diagrams with the same data:



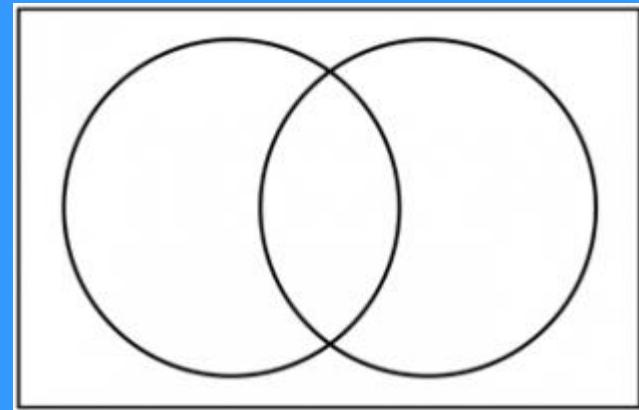
What other numbers could go in each region?

What is each region the set of?



Children should deconstruct a Carroll diagram and reconstruct it as a Venn diagram (& vice versa).

|  |                |            |
|--|----------------|------------|
|  |                |            |
|  | 7<br>35<br>56  | 140<br>490 |
|  | 18<br>24<br>51 | 30<br>500  |



They could also work out the captions for a completed Venn diagram then recreate it as a Carroll diagram.



**Link to useful resources from  
Nottingham's Primary Team:  
[goo.gl/Zwrj7I](http://goo.gl/Zwrj7I)**

**External links to other useful resources:**

<https://www.tes.com/teaching-resource/reasoning-and-problem-solving-questions-collection-ks1-and-ks2-11249968>

<https://www.thirdspacelearning.com/>

[jane.gill@nottinghamcity.gov.uk](mailto:jane.gill@nottinghamcity.gov.uk)

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